Closing Thu, Jan. 22: 12.6
Closing Tue, Jan. 27: 10.1/13.1, 10.2/13.2

12.6 A few basic 3D surfaces

First, a 2D review.

Line: $ax + by = c$

Parabola: $ax^2 + by = c$ or

$ax + by^2 = c$

Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

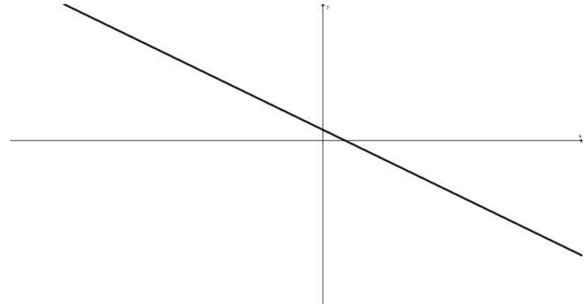
Hyperbola: $ax^2 - by^2 = c$ or

$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

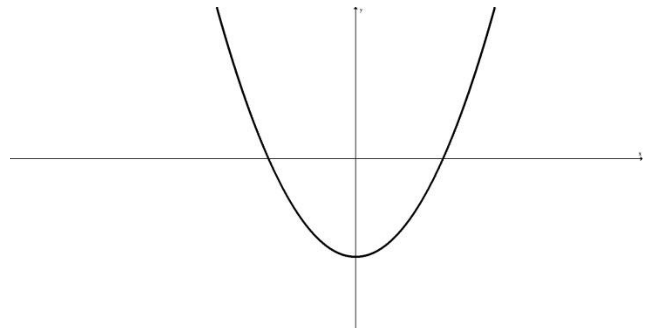
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Examples:

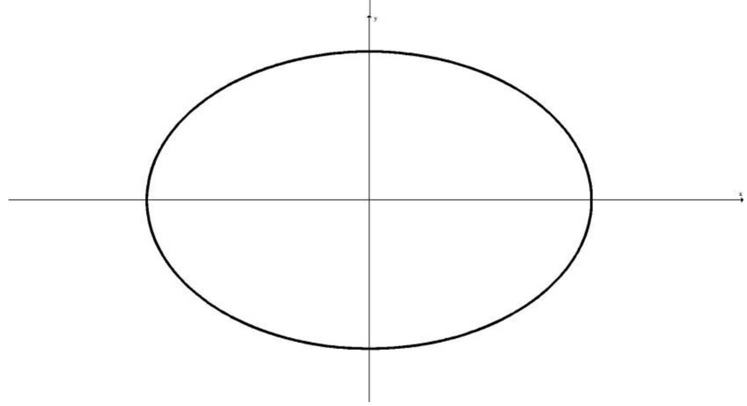
$3x + 2y = 1$



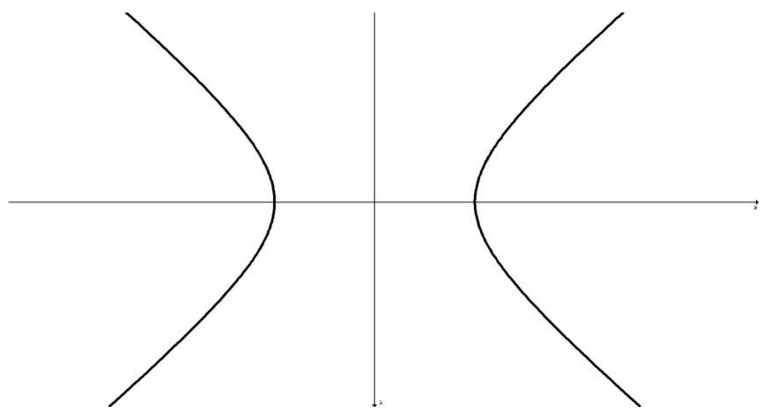
$3x^2 - y = 4$



$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$



$$\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$$



Now onto a few basic 3D Shapes

1. **Cylinders**: If one variable is absent, then the graph is a 2D curve extended into 3D.

   If the 2D shade is called "BLAH", then the 3D shade is called a "BLAH cylinder".

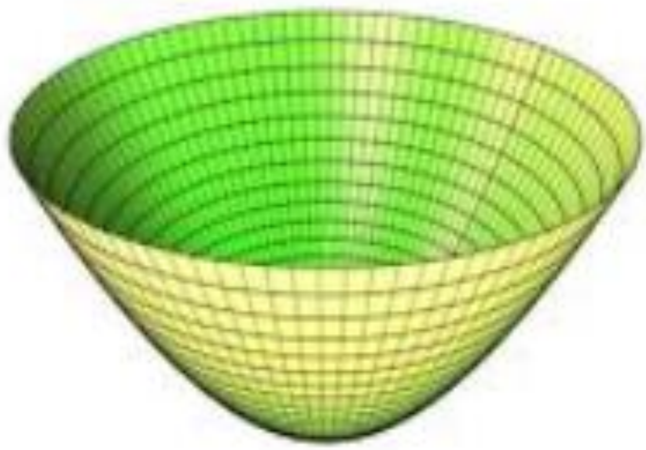
Examples:

(a) $x^2 + y^2 = 1$ in 3D is a circular cylinder (extended in the z-axis direction).

(b) $z = \cos(x)$ in 3D is a cosine cylinder (extended in the y-axis direction).

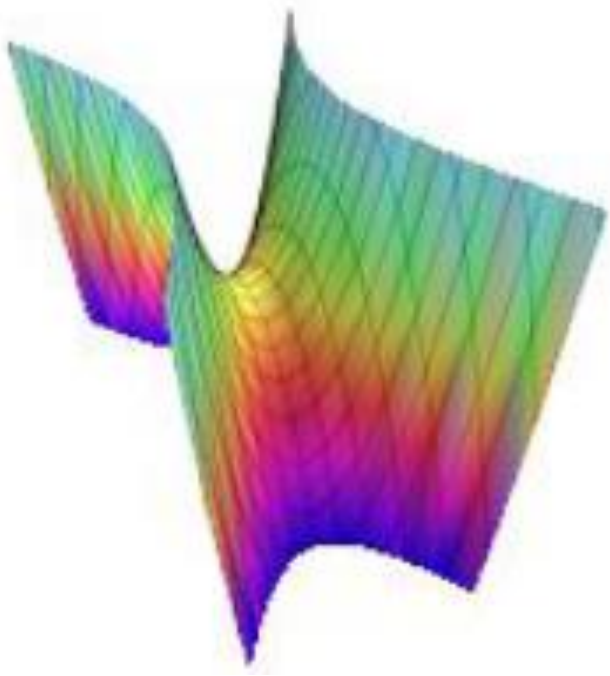
2. **Quadric Surfaces**: A surface given by an equation involving a sum of first and second powers of x, y, and z is called a quadric surface. To visualize, we use the concept of **traces**.

   We fix one variable and look at the resulting 2D picture (i.e. look at one slice). If we do several traces in different directions, we start to get an idea about the picture.



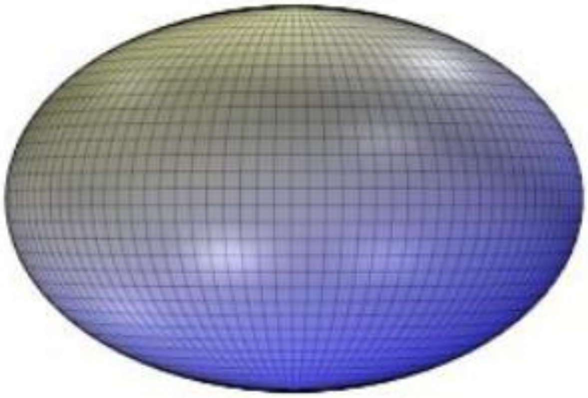
Elliptical Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$ (*ex*: $z = 3x^2 + 5y^2$)



Hyperbolic Paraboloid

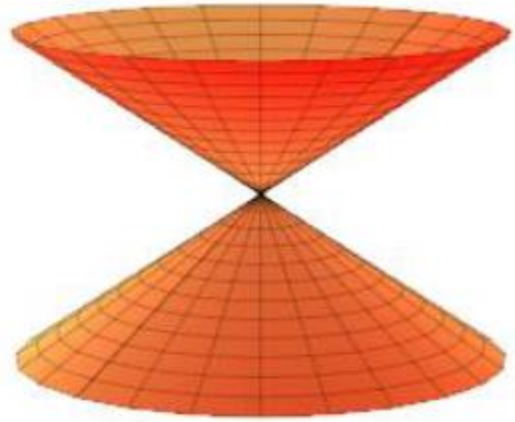
$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$ (*ex*: $y = 2x^2 - 5z^2$)



Ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

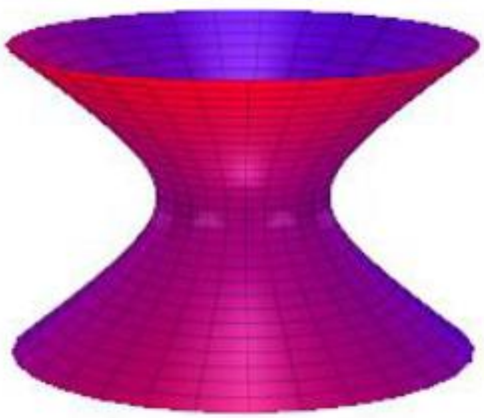
$(3x^2 + 5y^2 + z^2 = 3)$



Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

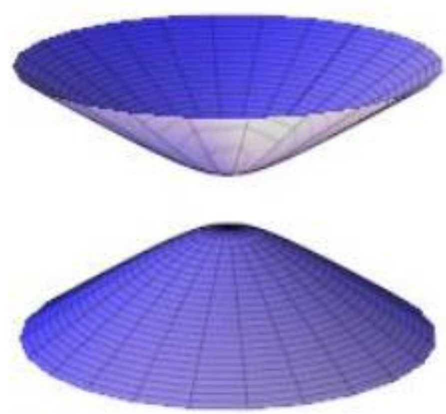
$(z^2 = x^2 + y^2)$



Hyperboloid of One Sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$(x^2 - y^2 + z^2 = 10)$



Hyperboloid of Two Sheets

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

$(x^2 + y^2 - z^2 = -4)$